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Monopole Excitation of the Nucleon in a Relativistic Three-Quark Model

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The densities and form factors of the proton and the Roper monopole excited-state resonance are calculated using a relativistic three-quark model. Small current quark masses are used with the three-body Dirac equation solved in hypercentral approximation. A QCD-based three-body potential, proportional to a minimum string length between the three quarks, is used for confinement. The calculated electric form factor for the proton reproduces closely a dipole fit to the data. The proton density is more compact than is the Roper resonance density. The central density of the proton is about five times that for the Roper resonance. The hyperradial node in the Roper resonance composite three-quark wave function shows up as a node in the transition density between the proton and the Roper resonance. This node also causes the calculated transition form factor to be larger than either the proton or Roper resonance form factors, all evaluated at the same value of momentum transfer. The Roper resonance form factor is smaller than the proton form factor, as expected, indicative of the Roper resonance being a more diffuse system than the proton.

1. INTRODUCTION

Among the excited states of the proton, the monopole or breathing mode collective excitations are distinguished by the fact that they correspond to radial oscillations in the density of the nucleon matter. The monopole state excitations characterize the nucleon compressibility. We study here the nature of the monopole excitations of the nucleon using a relativistic three-quark model. The masses of the u and d quarks will be taken as the same. Current quark masses for the up, down quarks in the nucleon are in the 2- to 10-

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MeV range [14] and the latter value is used here. Such quarks are extremely relativistic, even in the nucleon rest frame. The three-body Dirac equation is thereby used to describe the quark dynamics in the rest frame of the nucleon. The center-of-mass problem is properly handled using the hyperspherical coordinates method [17,7,6] in the rest frame of the system. This study is stimulated by the fact that the problem of identification of monopole resonances in experimental studies is not trivial, and for most baryons has not been solved uniquely. Therefore theoretical investigations which shed light on the nature of the wave functions and the various properties of monopole resonances are of interest. The method of hyperspherical functions is a very convenient basis for the microscopic description of monopole vibrations. In this approach, one introduces a collective variable, the hyperradius, which is associated with the mean square radius of the quark location within the nucleon. This can also be tied to the average density of a nucleon. Radial excitations of this variable correspond to monopole vibrations of the nucleon as a whole, and the density is a dynamic variable. The hyperspherical expansion utilizes a sum over configurations in describing the three-quark composite wave function. The hypercentral approximation truncates this expansion to one term. Here the quarks are assumed to be in a $(1/2^+)^3$ configuration, coupled to a total spin of 1/2 for either the proton or the Roper resonance. The space, spin color, and flavor dependences of the three-quark composite wave function are the same as in ref. 17. A scalar string flux tube potential [5,11] inspired by QCD considerations is used to model the quark interactions within the nucleon. The nucleon rest frame properties can be calculated using the composite three-quark wave function found from solving the three-body Dirac equation. It is:

$$\left[\begin{array}{c}(\vec{\alpha}_1 \cdot \vec{p}_1 + ms_1\beta_1) + (\vec{\alpha}_2 \cdot \vec{p}_2 + m_2\beta_2) \\ + (\alpha_3 \cdot p_3 + m_3\beta_3) + \beta_1\beta_2\beta_3 V\end{array}\right]\Psi = E\Psi \tag{1}$$

In our previous papers [17,18] we studied the properties of the nucleons within a relativistic three-quark context. The Ioffe coupling constant for the Roper resonance and for the proton has been calculated and compared with QCD results [8] and with the instanton approach [15]. In this paper we use the same approach to describe the form factors of the baryons. The form factors of the baryons have been studied [4] in the nonrelativistic potential model, where the importance of relativistic effects was shown. The detailed description of the baryon form factors within an algebraic collective model of the nucleons has been done by Bijker *et al.* [2]. Recently the exchange currents in the photoproduction of baryon resonances has been studied [12], as have the properties of the monopole excitations in relativistic models

[18,1]. The investigation of the monopole excitation in the solution of the Nambu–Jona–Lasinio soliton model has also been studied [1].

2. CALCULATIONAL DETAILS AND RESULTS

We use the $(1/2^+)^3$ configuration for the three quarks. The interaction is a scalar three-body stringlike potential, which for large quark separations is proportional to the distance between the three quarks. For smaller separations the potential is weaker and practically vanishes at short separations. This is modeled here by a negative constant term c,

$$V_{123} = bS - c \tag{2}$$

S is the minimum length between the quarks. The three quark locations always form a triangle. Providing the obtuse angle is less than 120 deg, the minimum length S becomes a Y-shaped string of three varying-length legs, each separated by 120 deg at the vertex. This potential is sometimes called a Mercedes Benz potential because this shape resembles the automobile logo. The string flux tube confining potential is independent of three of the hyperangles, depending only on the magnitude ratio r_1/r_2 and the angle between r_1 and r_2. The hypercentral approximation consists in replacing S by its hyperangular average and truncating the expansion over configurations to a single term. After averaging over hyperangles, the string length is proportional to the hyperradius. The string constant b is taken as 0.09 GeV^2, one-half the string constant deduced from quark–antiquark modeling of mesons [5,11,3,10]. The constant c is set to 0.2 GeV, so that the proton rest energy is reproduced with 10-MeV quark masses. Assuming the potential is color independent, one can integrate over the color part of the wave function, resulting in coupled first-order differential equations to solve for the hyperradial components of the composite three-quark wave function. This can be done numerically. This is an eigenvalue problem where the potential constant c is adjusted so that the components vanish asymptotically as the hyperradius becomes large. The ground-state proton wave function components have nodes only at the hyperradial origin and at infinity. The Roper resonance is taken to be the state where the components each have an additional hyperradial node. The eigenenergy for which this occurs, the predicted Roper energy 1.35 GeV, is smaller than experiment. This is determined using the same parameters as for the proton state.

The quark mass density in the system rest frame for both the proton and the Roper resonance states is shown in Fig. 1. Both are characterized by a monotonic decrease with a characteristic size of less than 1 fm. This is a success of this modeling using the hyperspherical method. The parameters were chosen to reproduce the rest mass of the proton, and the size comes

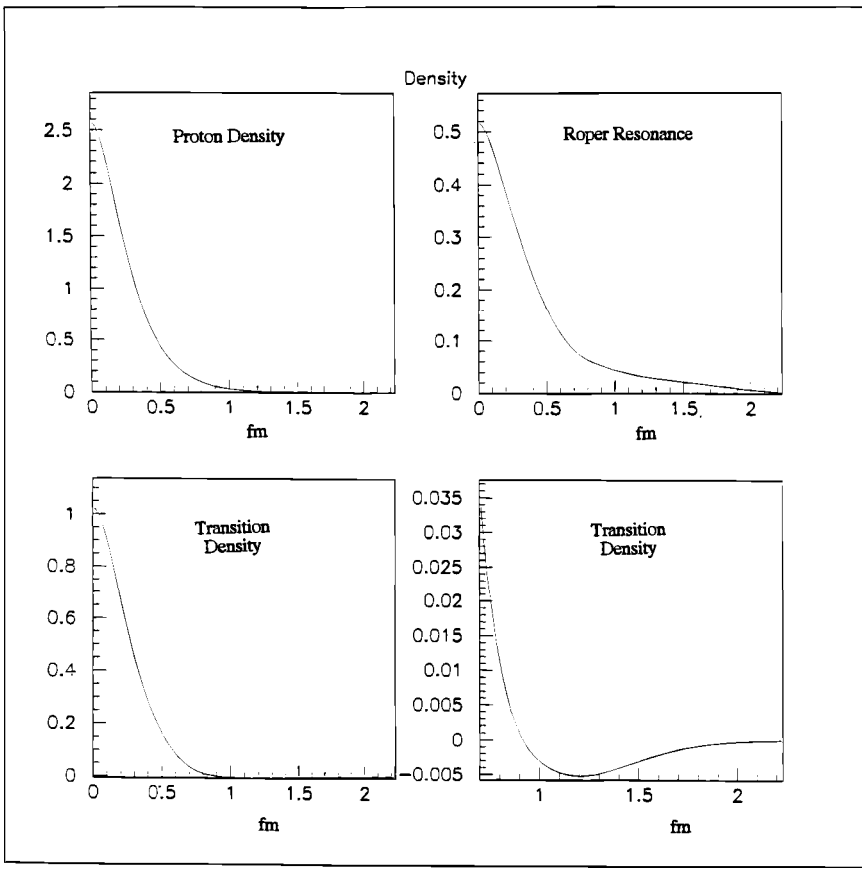

Fig. 1. The proton density, Roper resonance density, and their transition density versus distance from the center of mass. The details of the node in the transition density are shown in the lower right panel.

out quite well using only the $(1/2^+)^3$ configuration for the three quarks. The matter density at the center of the proton is about five times the calculated Roper resonance central density. The Roper resonance density has a tail out to about 2 fm radius that is not at all prominent for the proton case. Also shown in Fig. 1 is the proton-Roper resonance monopole transition density. The lower right panel has an expanded radial scale to show the details of a node in the transition density. This node can be traced to the hyperradial node in the Roper resonance wave function components since the proton wave function components have no hyperradial sign changes.

The electric form factor G_E for the proton versus q^2, the momentum transfer squared, can be seen in Fig. 2 as the dotted line. Also shown is the

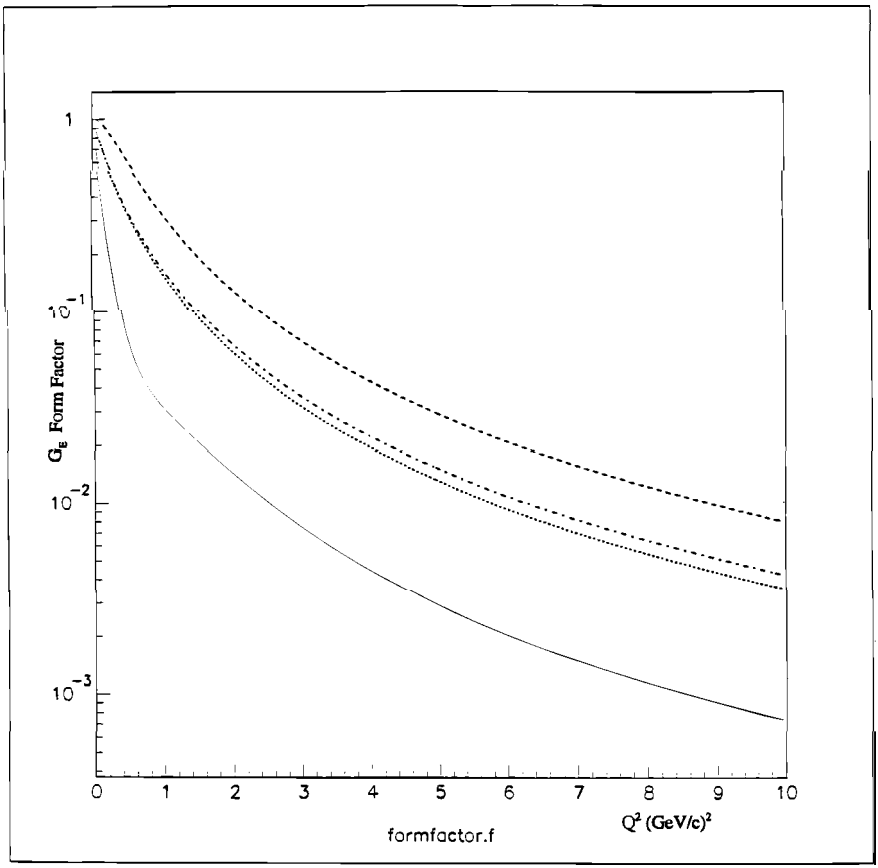

Fig. 2. G_E form factors versus momentum transfer q^2 in $(GeV/c)^2$. The dotted curve is for the proton. The solid curve is for the Roper resonance. The transition form factor is given by the dashed curve. The dipole fit for the proton is given by the dot-dashed curve. The form factors are calculated from the rest-frame form factors using the substitution method.

dipole form factor as the dot-dash line. The dipole form factor is an excellent fit to the experimental data for the proton electric form factor. The agreement between the calculated and dipole form factors is quite good. Also shown is the predicted electric form factor for the Roper resonance (solid line) and the electric transition form factor from the proton to the Roper resonance (dashed line). The electric form factor for the Roper resonance for a given value of momentum transfer lies below that for the proton. This is consistent with the proton being more compact spatially than is the Roper resonance. The transition form factor for a given momentum transfer lies above the form factors for both the proton and the Roper resonance. This is because some

of the oscillations in the $\sin(qr)$ factor of the integrand have the sign of their contribution reversed due to the node in the Roper resonance wave function, which appears linearly in the transition form factor. For both the proton and the Roper resonance form factors, the wave function appears squared, so the oscillations tend to cancel. These form factors are all calculated including the Lorentz contraction of the wave function along the direction of momentum transfer. We use the simple substitution law developed by Mitra and Kumari [13] and by Licht and Pagnamenta [9], where the form factors come from the rest-frame form factor as

$$F(q^2) = (M/E)^2 \, F_{rf}(M^2 q^2/E^2) \tag{3}$$

The Breit frame in the direction of momentum transfer is Lorentz-contracted by the inverse boost factor of

$$M/E = (1 + q^2/4M^2)^{-1/2} \tag{4}$$

In Fig. 3 the calculated G_E/dipole ratio of form factors is compared to the ratio of the experimental G_E/dipole ratio of form factors. The calculated ratio is lower than the experimental ratio, but generally falls within the experimental error bars. The calculated proton form factor has adequately reproduced experimental data. This model predicts a form factor for the Roper resonance that is more sharply peaked at small momentum transfers. For larger momentum transfers, the predicted Roper resonance form factor is an order of magnitude smaller than for the proton.

The densities and form factors of the proton and the monopole excited Roper resonance state have been calculated using a relativistic three-quark model. Small current quark masses are used with the three-body Dirac equation solved in hypercentral approximation. A QCD-based three-body potential proportional to a minimum string length between the three quarks is used for confinement. The calculated electric form factor for the proton reproduces closely a dipole fit to the data. The proton density is more compact than is the Roper resonance density. The central density of the proton is about five times that for the Roper resonance. The hyperradial node in the Roper resonance wave function shows up as a node in the transition density between the proton and the Roper resonance. This node also causes the calculated transition form factor to be larger than either the proton or Roper resonance form factors, all evaluated at the same value of momentum transfer. The Roper resonance form factor is smaller than the proton form factor, as expected, indicative of the Roper resonance being a more spatially diffuse system than is the proton.

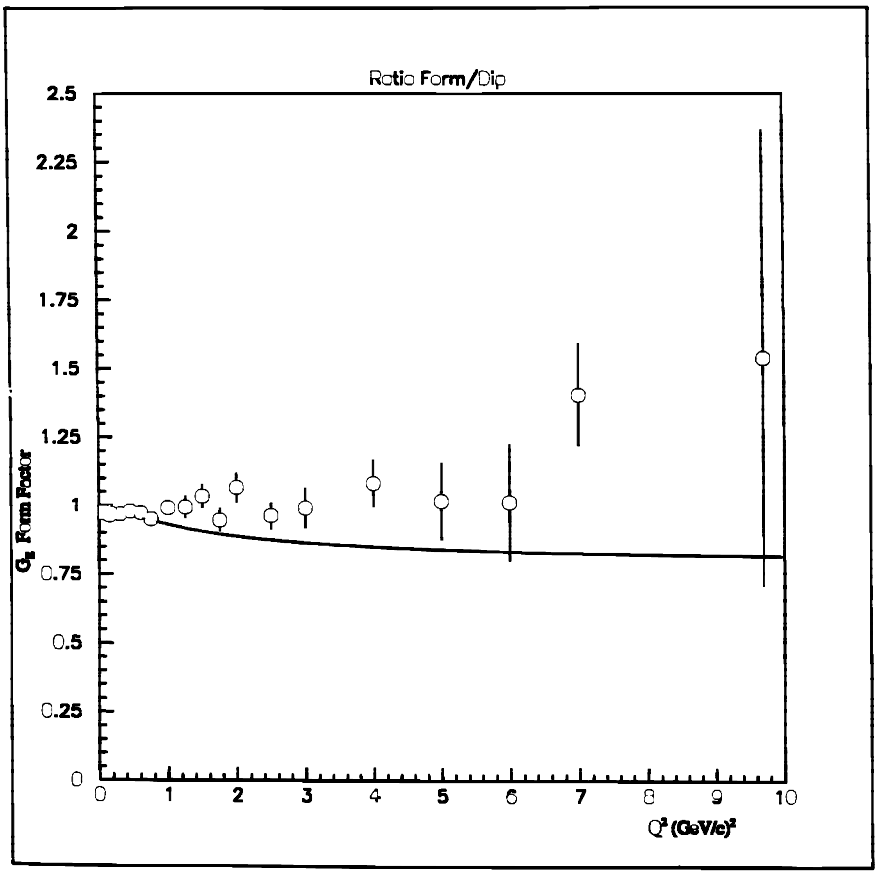

Fig. 3. Ratio of the calculated G_E/dipole form factors compared to the experimental form factor/dipole ratio for the proton, using the relativistic quark model.

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